Overview of the Integrated Pathway for the Common Core State Mathematics Standards

This table shows the domains and clusters in each course in the Integrated Pathway. The standards from each cluster included in that course are listed below each cluster. For each course, limits and focus for the clusters are shown in italics.

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses*
Number and Quantity	The Real Number System		• Extend the properties of exponents to rational exponents. N.RN.1, 2 • Use properties of rational and irrational numbers. N.RN.3		
	Quantities	• Reason quantitatively and use units to solve problems. *Foundation for work with expressions, equations and functions* N.Q.1, 2, 3			
	The Complex Number System		• Perform arithmetic operations with complex numbers. *i^2 as highest power of i* N.CN.1, 2 • Use complex numbers in polynomial identities and equations. *Quadratics with real coefficients* N.CN.7, (+) 8, (+) 9	• Use complex numbers in polynomial identities and equations. *Polynomials with real coefficients; apply N.CN.9 to higher degree polynomials* (+) N.CN. 8, 9	• Perform arithmetic operations with complex numbers. (+) N.CN.3 • Represent complex numbers and their operations on the complex plane. (+) N.CN.4, 5, 6
	Vector Quantities and Matrices				• Represent and model with vector quantities. (+) N.VM.1, 2, 3 • Perform operations on vectors. (+) N.VM.4a, 4b, 4c, 5a, 5b • Perform operations on matrices and use matrices in applications. (+) N.VM.6, 7, 8, 9, 10, 11, 12

*The (+) standards in this column are those in the Common Core State Standards that are not included in any of the Integrated Pathway courses. They would be used in additional courses developed to follow Mathematics III.

Algebra

Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Seeing Structure in Expressions	• Interpret the structure of expressions. *Linear expressions and exponential expressions with integer exponents* A.SSE.1a, 1b	• Interpret the structure of expressions. *Quadratic and exponential* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. *Quadratic and exponential* A.SSE.3a, 3b, 3c	• Interpret the structure of expressions. *Polynomial and rational* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. A.SSE.4	
Arithmetic with Polynomials and Rational Expressions		• Perform arithmetic operations on polynomials. *Polynomials that simplify to quadratics* A.APR.1	• Perform arithmetic operations on polynomials. *Beyond quadratic* A.APR.1 • Understand the relationship between zeros and factors of polynomials. A.APR.2, 3 • Use polynomial identities to solve problems. A.APR.4, (+) 5 • Rewrite rational expressions. *Linear and quadratic denominators* A.APR.6, (+) 7	
Creating Equations	• Create equations that describe numbers or relationships. *Linear, and exponential (integer inputs only); for A.CED.3, linear only* A.CED. 1, 2, 3, 4	• Create equations that describe numbers or relationships. *In A.CED.4, include formulas involving quadratic terms* A.CED. 1, 2, 4	• Create equations that describe numbers or relationships. *Equations using all available types of expressions including simple root functions* A.CED.1, 2, 3, 4	

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Algebra	Reasoning with Equations and Inequalities	• Understand solving equations as a process of reasoning and explain the reasoning. *Master linear, learn as general principle* A.REI.1 • Solve equations and inequalities in one variable. *Linear inequalities; literal that are linear in the variables being solved for;, exponential of a form, such as $2^x = 1/16$* A.REI.3 • Solve systems of equations. *Linear systems* A.REI.5, 6 • Represent and solve equations and inequalities graphically. *Linear and exponential; learn as general principle* A.REI.10, 11, 12	• Solve equations and inequalities in one variable. *Quadratics with real coefficients* A.REI.4a, 4b • Solve systems of equations. *Linear-quadratic systems* A.REI.7	• Understand solving equations as a process of reasoning and explain the reasoning. *Simple radical and rational* A.REI.2 • Represent and solve equations and inequalities graphically. *Combine polynomial, rational, radical, absolute value, and exponential functions* A.REI.11	• Solve systems of equations. (+) A.REI.8, 9
Functions	Interpreting Functions	• Understand the concept of a function and use function notation. *Learn as general principle. Focus on linear and exponential (integer domains) and on arithmetic and geometric sequences* F.IF.1, 2, 3 • Interpret functions that arise in applications in terms of a context. *Linear and exponential, (linear domain)* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear and exponential* F.IF.7a, 7e, 9	• Interpret functions that arise in applications in terms of a context. *Quadratic* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear, exponential, quadratic, absolute value, step, piecewise-defined* F.IF.7a, 7b, 8a, 8b, 9	• Interpret functions that arise in applications in terms of a context. *Include rational, square root and cube root; emphasize selection of appropriate models* F.IF.4, 5, 6 • Analyze functions using different representations. *Include rational and radical; focus on using key features to guide selection of appropriate type of model function* F.IF. 7b, 7c, 7e, 8, 9	• Analyze functions using different representations. *Logarithmic and trigonometric functions* (+) F.IF.7d

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Functions	Building Functions	• Build a function that models a relationship between two quantities. *For F.BF.1, 2, linear and exponential (integer inputs)* F.BF.1a, 1b, 2 • Build new functions from existing functions. *Linear and exponential; focus on vertical translations for exponential* F.BF.3	• Build a function that models a relationship between two quantities. *Quadratic and exponential* F.BF.1a, 1b • Build new functions from existing functions. *Quadratic, absolute value* F.BF.3, 4a	• Build a function that models a relationship between two quantities. *Include all types of functions studied* F.BF.1b • Build new functions from existing functions. *Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types* F.BF.3, 4a	• Build a function that models a relationship between two quantities. (+) F.BF.1c • Build new functions from existing functions. (+) F.BF.4b, 4c, 4d, 5
	Linear, Quadratic, and Exponential Models	• Construct and compare linear, quadratic, and exponential models and solve problems. *Linear and exponential* F.LE.1a, 1b, 1c, 2, 3 • Interpret expressions for functions in terms of the situation they model. *Linear and exponential of form* $f(x) = b^x + k$ F.LE.5	• Construct and compare linear, quadratic, and exponential models and solve problems. *Include quadratic* F.LE. 3	• Construct and compare linear, quadratic, and exponential models and solve problems. *Logarithms as solutions for exponentials* F.LE.4	
	Trigonometric Functions		• Prove and apply trigonometric identities. F.TF.8	• Extend the domain of trigonometric functions using the unit circle. F.TF.1, 2 • Model periodic phenomena with trigonometric functions. F.TF. 5	• Extend the domain of trigonometric functions using the unit circle. (+) F.TF.3, 4 • Model periodic phenomena with trigonometric functions. (+) F.TF. 6, 7 • Prove and apply trigonometric identities. (+) F.TF. 9

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Geometry	Congruence	• Experiment with transformations in the plane. G.CO.1, 2, 3, 4, 5 • Understand congruence in terms of rigid motions. *Build on rigid motions as a familiar starting point for development of concept of geometric proof* G.CO.6, 7, 8 • Make geometric constructions. *Formalize and explain processes* G.CO.12, 13	• Prove geometric theorems. *Focus on validity of underlying reasoning while using variety of ways of writing proofs* G.CO.9, 10, 11		
	Similarity, Right Triangles, and Trigonometry		• Understand similarity in terms of similarity transformations. G.SRT.1a, 1b, 2, 3 • Prove theorems involving similarity. *Focus on validity of underlying reasoning while using variety of formats* G.SRT.4, 5 • Define trigonometric ratios and solve problems involving right triangles. G.SRT.6, 7, 8	• Apply trigonometry to general triangles. (+) G.SRT.9. 10, 11	
	Circles		• Understand and apply theorems about circles. G.C.1, 2, 3, (+) 4 • Find arc lengths and areas of sectors of circles. *Radian introduced only as unit of measure* G.C.5		

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Geometry	Expressing Geometric Properties with Equations	• Use coordinates to prove simple geometric theorems algebraically. *Include distance formula; relate to Pythagorean theorem* G.GPE. 4, 5, 7	• Translate between the geometric description and the equation for a conic section. G.GPE.1, 2 • Use coordinates to prove simple geometric theorems algebraically. *For G.GPE.4 include simple circle theorems* G.GPE.4		• Translate between the geometric description and the equation for a conic section. (+) G.GPE.3
	Geometric Measurement and Dimension		• Explain volume formulas and use them to solve problems. G.GMD.1, 3	• Visualize the relation between two-dimensional and three-dimensional objects. G.GMD.4	• Explain volume formulas and use them to solve problems. (+) G.GMD.2
	Modeling with Geometry			• Apply geometric concepts in modeling situations. G.MG.1, 2, 3	
Statistics and Probability	Interpreting Categorical and Quantitative Data	• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.1, 2, 3 • Summarize, represent, and interpret data on two categorical and quantitative variables. *Linear focus; discuss general principle* S.ID.5, 6a, 6b, 6c • Interpret linear models. S.ID.7, 8, 9		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.4	
	Making Inferences and Justifying Conclusions			• Understand and evaluate random processes underlying statistical experiments. S.IC.1, 2 • Make inferences and justify conclusions from sample surveys, experiments and observational studies. S.IC.3, 4, 5, 6	

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Statistics and Probability	Conditional Probability and the Rules of Probability		• Understand independence and conditional probability and use them to interpret data. *Link to data from simulations or experiments* S.CP.1, 2, 3, 4, 5 • Use the rules of probability to compute probabilities of compound events in a uniform probability model. S.CP.6, 7, (+) 8, (+) 9		
	Using Probability to Make Decisions		• Use probability to evaluate outcomes of decisions. *Introductory; apply counting rules* (+) S.MD.6, 7	• Use probability to evaluate outcomes of decisions. *Include more complex situations* (+) S.MD.6, 7	• Calculate expected values and use them to solve problems. (+) S.MD.1, 2, 3, 4 • Use probability to evaluate outcomes of decisions. (+) S.MD. 5a, 5b

Integrated Pathway: Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics 1 uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Critical Area 4: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 6: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Relationships Between Quantities</td><td>Reason quantitatively and use units to solve problems.
Interpret the structure of expressions.
Create equations that describe numbers or relationships.</td><td rowspan="6">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Linear and Exponential Relationships</td><td>Represent and solve equations and inequalities graphically.
Understand the concept of a function and use function notation.
Interpret functions that arise in applications in terms of a context.
Analyze functions using different representations.
Build a function that models a relationship between two quantities.
Build new functions from existing functions.
Construct and compare linear, quadratic, and exponential models and solve problems.
Interpret expressions for functions in terms of the situation they model.</td></tr>
<tr><td>Unit 3†
Reasoning with Equations</td><td>Understand solving equations as a process of reasoning and explain the reasoning.
Solve equations and inequalities in one variable.
Solve systems of equations.</td></tr>
<tr><td>Unit 4
Descriptive Statistics</td><td>Summarize, represent, and interpret data on a single count or measurement variable.
Summarize, represent, and interpret data on two categorical and quantitative variables.
Interpret linear models.</td></tr>
<tr><td>Unit 5
Congruence, Proof, and Constructions</td><td>Experiment with transformations in the plane.
Understand congruence in terms of rigid motions.
Make geometric constructions.</td></tr>
<tr><td>Unit 6
Connecting Algebra and Geometry through Coordinates</td><td>Use coordinates to prove simple geometric theorems algebraically.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

†Note that solving equations and systems of equations follows a study of functions in this course. To examine equations before functions, this unit could be merged with Unit 1.

Unit 1: Relationships Between Quantities

By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Unit 1: Relationships between Quantities	
Clusters with Instructional Notes	**Common Core State Standards**
SKILLS TO MAINTAIN *Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.*	
• Reason quantitatively and use units to solve problems. *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*	N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
• Interpret the structure of expressions. *Limit to linear expressions and to exponential expressions with integer exponents.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
• Create equations that describe numbers or relationships. *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas with a linear focus.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Unit 2: Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Represent and solve equations and inequalities graphically. *For A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where f(x) and g(x) are linear or exponential.*	A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
• Understand the concept of a function and use function notation. *Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.* *Draw examples from linear and exponential functions. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.*	F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret functions that arise in applications in terms of a context. *For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.* *N.RN.1 and N.RN. 2 will need to be referenced here before discussing exponential models with continuous domains.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100\cdot2^n$.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
• Build a function that models a relationship between two quantities. *Limit F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
• Build new functions from existing functions. *Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.* *While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Construct and compare linear, quadratic, and exponential models and solve problems. *For F.LE.3, limit to comparisons between exponential and linear models.*	F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
• Interpret expressions for functions in terms of the situation they model. *Limit exponential functions to those of the form $f(x) = b^x + k$.*	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Unit 3: Reasoning with Equations

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Unit 3: Reasoning with Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Understand solving equations as a process of reasoning and explain the reasoning. *Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.*	A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
• Solve equations and inequalities in one variable. *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.*	A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
• Solve systems of equations. *Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5, which requires students to prove the slope criteria for parallel lines.*	A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Unit 4: Descriptive Statistics

Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatterplots and recognizing linear trends in data. This unit builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 4: Descriptive Statistics	
Clusters with Instructional Notes	**Common Core State Standards**
• Summarize, represent, and interpret data on a single count or measurement variable. *In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.*	S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
• Summarize, represent, and interpret data on two categorical and quantitative variables. *Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.* *S.ID.6b should be focused on situations for which linear models are appropriate.*	S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for scatter plots that suggest a linear association.
• Interpret linear models. *Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.*	S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.9 Distinguish between correlation and causation.

Unit 5: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Unit 5: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	**Common Core State Standards**
• Experiment with transformations in the plane. *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.*	G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
• Understand congruence in terms of rigid motions. *Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.*	G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
• Make geometric constructions. *Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects.* *Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.*	G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Unit 6: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Unit 6: Connecting Algebra and Geometry Through Coordinates	
Clusters and Instructional Notes	**Common Core State Standards**
• Use coordinates to prove simple geometric theorems algebraically. *This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPE.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.* *Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in Mathematics I involving systems of equations having no solution or infinitely many solutions.* *G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean theorem.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).* G.GPE.5 Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Integrated Pathway: Mathematics II

The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I as organized into 6 critical areas, or units. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 3, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

Critical Area 2: Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Critical Area 3: Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

Critical Area 4: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Critical Area 5: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

Critical Area 6: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Extending the Number System</td><td>• Extend the properties of exponents to rational exponents.
• Use properties of rational and irrational numbers.
• Perform arithmetic operations with complex numbers.
• Perform arithmetic operations on polynomials.</td><td rowspan="5">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Quadratic Functions and Modeling</td><td>• Interpret functions that arise in applications in terms of a context.
• Analyze functions using different representations.
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.
• Construct and compare linear, quadratic, and exponential models and solve problems.</td></tr>
<tr><td>Unit 3†
Expressions and Equations</td><td>• Interpret the structure of expressions.
• Write expressions in equivalent forms to solve problems.
• Create equations that describe numbers or relationships.
• Solve equations and inequalities in one variable.
• Use complex numbers in polynomial identities and equations.
• Solve systems of equations.</td></tr>
<tr><td>Unit 4
Applications of Probability</td><td>• Understand independence and conditional probability and use them to interpret data.
• Use the rules of probability to compute probabilities of compound events in a uniform probability model.
• Use probability to evaluate outcomes of decisions.</td></tr>
<tr><td>Unit 5
Similarity, Right Triangle Trigonometry, and Proof</td><td>• Understand similarity in terms of similarity transformations.
• Prove geometric theorems.
• Prove theorems involving similarity.
• Use coordinates to prove simple geometric theorems algebraically.
• Define trigonometric ratios and solve problems involving right triangles.
• Prove and apply trigonometric identities.</td></tr>
<tr><td>Unit 6
Circles With and Without Coordinates</td><td>• Understand and apply theorems about circles.
• Find arc lengths and areas of sectors of circles.
• Translate between the geometric description and the equation for a conic section.
• Use coordinates to prove simple geometric theorem algebraically.
• Explain volume formulas and use them to solve problems.</td><td></td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

†Note that solving equations follows a study of functions in this course. To examine equations before functions, this unit could come before Unit 2.

Unit 1: Extending the Number System

Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 2, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

Unit 1: Extending the Number System	
Clusters with Instructional Notes	**Common Core State Standards**
• Extend the properties of exponents to rational exponents.	N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
• Use properties of rational and irrational numbers. *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.*	N.RN.3 Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.
• Perform arithmetic operations with complex numbers. *Limit to multiplications that involve i^2 as the highest power of i.*	N.CN.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real. N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
• Perform arithmetic operations on polynomials. *Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.*	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Unit 2: Quadratic Functions and Modeling

Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Unit 2: Quadratic Functions and Modeling	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret functions that arise in applications in terms of a context. *Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Mathematics I on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic.* *Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
• Build a function that models a relationship between two quantities. *Focus on situations that exhibit a quadratic or exponential relationship.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Unit 2: Quadratic Functions and Modeling	
Clusters with Instructional Notes	**Common Core State Standards**
• Build new functions from existing functions. *For F.BF.3, focus on quadratic functions and consider including absolute value functions.. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x>0$.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2\,x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*
• Construct and compare linear, quadratic, and exponential models and solve problems. *Compare linear and exponential growth studied in Mathematics I to quadratic growth.*	F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Unit 3: Expressions and Equations

Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

Unit 3: Expressions and Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret the structure of expressions. *Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
• Write expressions in equivalent forms to solve problems. *It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.*	A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
• Create equations that describe numbers or relationships. *Extend work on linear and exponential equations in Mathematics I to quadratic equations. Extend A.CED.4 to formulas involving squared variables.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*
• Solve equations and inequalities in one variable. *Extend to solving any quadratic equation with real coefficients, including those with complex solutions.*	A.REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
• Use complex numbers in polynomial identities and equations. *Limit to quadratics with real coefficients.*	N.CN.7 Solve quadratic equations with real coefficients that have complex solutions. N.CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Unit 3: Expressions and Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Solve systems of equations. *Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.*	A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

Unit 4: Applications of Probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Unit 4: Applications of Probability	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand independence and conditional probability and use them to interpret data. *Build on work with two-way tables from Mathematics I Unit 4 (S.ID.5) to develop understanding of conditional probability and independence.*	S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.3 Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
• Use the rules of probability to compute probabilities of compound events in a uniform probability model.	S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. S.CP.7 Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A* and *B)*, and interpret the answer in terms of the model. S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, *P(A* and *B) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model. S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
• Use probability to evaluate outcomes of decisions. *This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.*	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Unit 5: Similarity, Right Triangle Trigonometry, and Proof

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem.

It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

Unit 5: Similarity, Right Triangle Trigonometry, and Proof	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand similarity in terms of similarity transformations.	G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
• Prove geometric theorems. *Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 6.*	G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*
• Prove theorems involving similarity.	G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
• Use coordinates to prove simple geometric theorems algebraically.	G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
• Define trigonometric ratios and solve problems involving right triangles.	G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Unit 5: Similarity, Right Triangle Trigonometry, and Proof	
Clusters and Instructional Notes	**Common Core State Standards**
• Prove and apply trigonometric identities. *In this course, limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. A course with a greater focus on trigonometry could include the (+) standard F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could continue to be limited to acute angles in Mathematics II.* *Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.*	F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.

Unit 6: Circles With and Without Coordinates

In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Unit 6: Circles With an Without Coordinates	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand and apply theorems about circles.	G.C.1 Prove that all circles are similar. G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.
• Find arc lengths and areas of sectors of circles. *Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.*	G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
• Translate between the geometric description and the equation for a conic section. *Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.*	G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.2 Derive the equation of a parabola given a focus and directrix.
• Use coordinates to prove simple geometric theorems algebraically. *Include simple proofs involving circles.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
• Explain volume formulas and use them to solve problems. *Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.*	G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Integrated Pathway: Mathematics III

It is in Mathematics III that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions.[3] They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Critical Area 2: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 3: Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles open up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

Critical Area 4: In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

[3]In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Inferences and Conclusions from Data</td><td>• Summarize, represent, and interpret data on single count or measurement variable.
• Understand and evaluate random processes underlying statistical experiments.
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
• Use probability to evaluate outcomes of decisions.</td><td rowspan="4">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Polynomial, Rational, and Radical Relationships.</td><td>• Use complex numbers in polynomial identities and equations.
• Interpret the structure of expressions.
• Write expressions in equivalent forms to solve problems.
• Perform arithmetic operations on polynomials.
• Understand the relationship between zeros and factors of polynomials.
• Use polynomial identities to solve problems.
• Rewrite rational expressions.
• Understand solving equations as a process of reasoning and explain the reasoning.
• Represent and solve equations and inequalities graphically.
• Analyze functions using different representations.</td></tr>
<tr><td>Unit 3
Trigonometry of General Triangles and Trigonometric Functions</td><td>• Apply trigonometry to general triangles.
• Extend the domain of trigonometric functions using the unit circle.
• Model periodic phenomena with trigonometric function.</td></tr>
<tr><td>Unit 4
Mathematical Modeling</td><td>• Create equations that describe numbers or relationships.
• Interpret functions that arise in applications in terms of a context.
• Analyze functions using different representations.
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.
• Construct and compare linear, quadratic, and exponential models and solve problems.
• Visualize relationships between two-dimensional and three-dimensional objects.
• Apply geometric concepts in modeling situations.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Unit 1: Inferences and Conclusions from Data

In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Unit 1: Inferences and Conclusions from Data	
Clusters and Instructional Notes	**Common Core State Standards**
• Summarize, represent, and interpret data on a single count or measurement variable. *While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.*	S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
• Understand and evaluate random processes underlying statistical experiments. *For S.IC.2, include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.*	S.IC.1 Understand that statistics allows inferences to be made about population parameters based on a random sample from that population. S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies. *In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons., These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.* *For S.IC.4 and 5, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.*	S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.6 Evaluate reports based on data.
• Use probability to evaluate outcomes of decisions. *Extend to more complex probability models. Include situations such as those involving quality control or diagnostic tests that yields both false positive and false negative results.*	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Unit 2: Polynomials, Rational, and Radical Relationships

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Unit 2: Polynomials, Rational, and Radical Relationships	
Clusters and Instructional Notes	**Common Core State Standards**
• Use complex numbers in polynomial identities and equations. *Build on work with quadratics equations in Mathematics II. Limit to polynomials with real coefficients.*	N.CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
• Interpret the structure of expressions. *Extend to polynomial and rational expressions.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
• Write expressions in equivalent forms to solve problems. *Consider extending A.SSE.4 to infinite geometric series in curricular implementations of this course description.*	A.SSE.4 Derive the formula for the sum of a geometric series (when the common ratio is not 1) , and use the formula to solve problems. *For example, calculate mortgage payments.* ★
• Perform arithmetic operations on polynomials. *Extend beyond the quadratic polynomials found in Mathematics II.*	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
• Understand the relationship between zeros and factors of polynomials.	A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Unit 2: Polynomials, Rational, and Radical Relationships	
Clusters and Instructional Notes	**Common Core State Standards**
• Use polynomial identities to solve problems. *This cluster has many possibilities for optional enrichment, such as relating the example in A.APR.4 to the solution of the system $u^2+v^2=1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x+y)^{n+1} = (x+y)(x+y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.*	A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.* A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
• Rewrite rational expressions *The limitations on rational functions apply to the rational expressions in A.APR.6. A.APR.7 requires the genera division algorithm for polynomials.*	A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
• Understand solving equations as a process of reasoning and explain the reasoning. *Extend to simple rational and radical equations.*	A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
• Represent and solve equations and inequalities graphically. *Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.*	A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
• Analyze functions using different representations. *Relate F.IF.7c to the relationship between zeros of quadratic functions and their factored forms.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Unit 3: Trigonometry of General Triangles and Trigonometric Functions

Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles open up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

Unit 3: Trigonometry of General Triangles and Trigonometric Functions	
Clusters and Instructional Notes	**Common Core State Standards**
• Apply trigonometry to general triangles. *With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.*	G.SRT.9 (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
• Extend the domain of trigonometric functions using the unit circle.	F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
• Model periodic phenomena with trigonometric functions.	F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

Unit 4: Mathematical Modeling

In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Unit 4: Mathematical Modeling	
Clusters and Instructional Notes	**Common Core State Standards**
• Create equations that describe numbers or relationships. *For A.CED.1, use all available types of functions to create such equations, including root functions, but constrain to simple cases. While functions used in A.CED.2, 3, and 4 will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Mathematics I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. Note that the example given for A.CED.4 applies to earlier instances of this standard, not to the current course.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
• Interpret functions that arise in applications in terms of a context. *Emphasize the selection of a model function based on behavior of data and context.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Unit 4: Mathematical Modeling	
Clusters and Instructional Notes	**Common Core State Standards**
• Build a function that models a relationship between two quantities. *Develop models for more complex or sophisticated situations than in previous courses.*	F.BF.1 Write a function that describes a relationship between two quantities.* b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
• Build new functions from existing functions. *Use transformations of functions to find more optimum models as students consider increasingly more complex situations.* *For F.BF.3, note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by a graph.* *Extend F.BF.4a to simple rational, simple radical, and simple exponential functions; connect F.BF.4a to F.LE.4.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2\,x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
• Construct and compare linear, quadratic, and exponential models and solve problems. *Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x + log y.*	F.LE.4 For exponential models, express as a logarithm the solution to $a\,b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
• Visualize relationships between two-dimensional and three-dimensional objects.	G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
• Apply geometric concepts in modeling situations.	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★ G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★ G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★